ARi Amalgamated Research Inc. www.arifractal.com

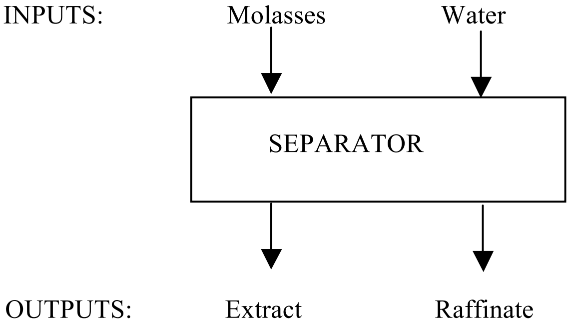
THE USE OF INFINITE SERIES FOR OPTIMIZING PLACEMENT AND OPERATION OF CHROMATOGRAPHIC SEPARATORS

Mike Kearney

Chromatographic separators are used by the sugar industry for the recovery of sucrose from impure solutions. Placement of a chromatographic separator on the sugar end generally results in some type of recycle loop. The equilibrium recycle conditions can be modeled using infinite series. The series limit provides a frame of reference for determining maximum product sucrose recovery, separator loading, evaporation requirements, etc. Individual series terms aid in determining the approach to the limit and suggest advantageous operating conditions.

The Separator

Although a separator can be used on nearly any stream of a sugar factory, economic factors usually dictate using molasses. In general, the separator operation is simple:



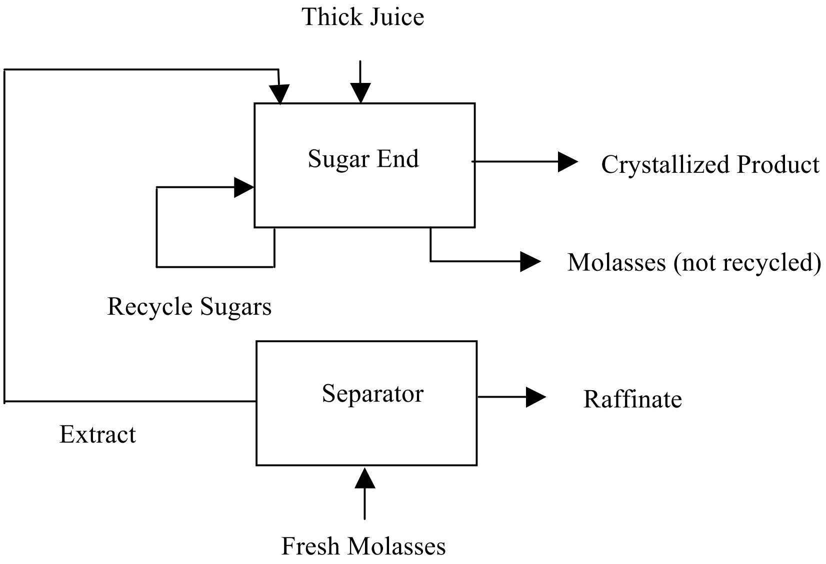
The feed materials consist of only molasses and water. Extract contains the recovered sucrose at high purity. This material is usually recycled in some way to the factory for eventual crystallization. The raffinate contains the molasses non-sugar and some sucrose. It is usually brixed up and used as a feed supplement or pulp additive.

Some highlights of the separator process relative to other sucrose recovery systems are:

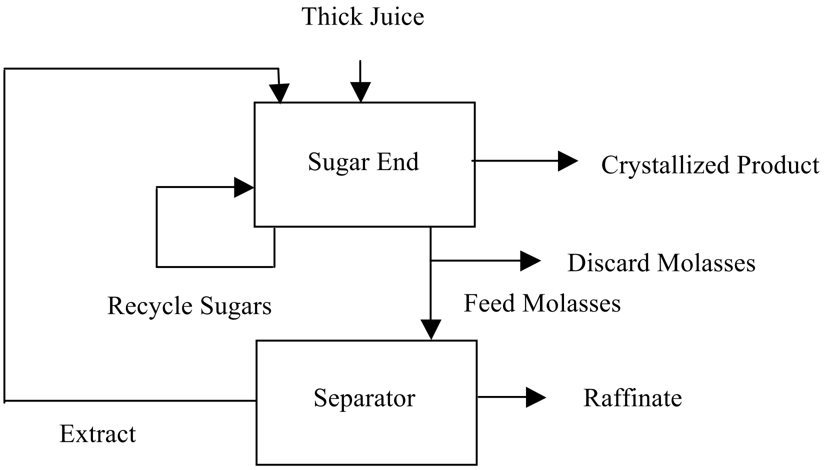
- High sucrose recoveries are possible
- High extract purities are possible
- Color and raffinose removal can be very good
- No special chemicals are added to the process (only molasses and water)

There are several ways to integrate a separator into the sugar end. For a molasses separator, the following are typical:

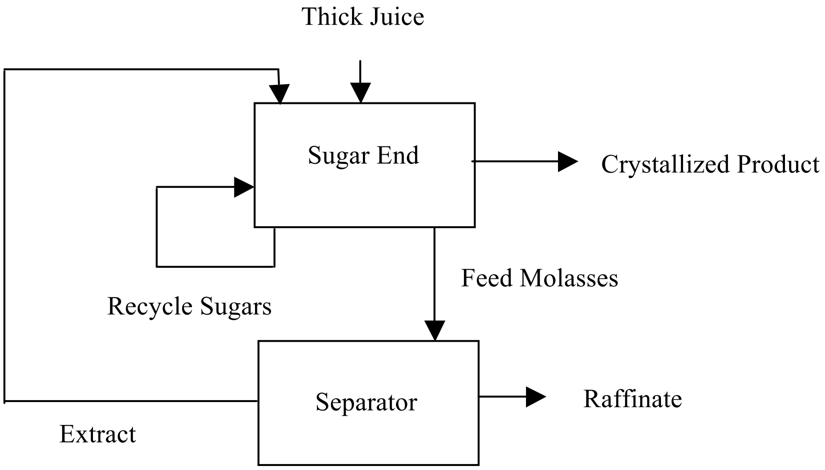
(1) No molasses recycle



(2) Molasses recycle with discard



(3) Complete recycle

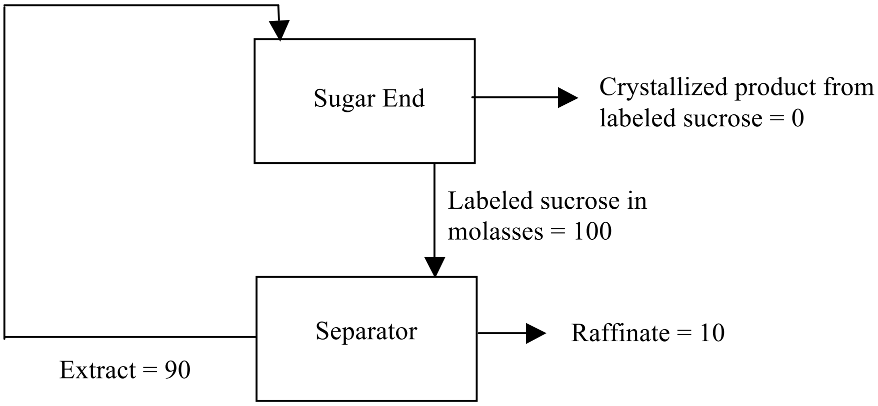


(4) Combinations of the above can be used, for example, operating different configurations during specific periods of the year.

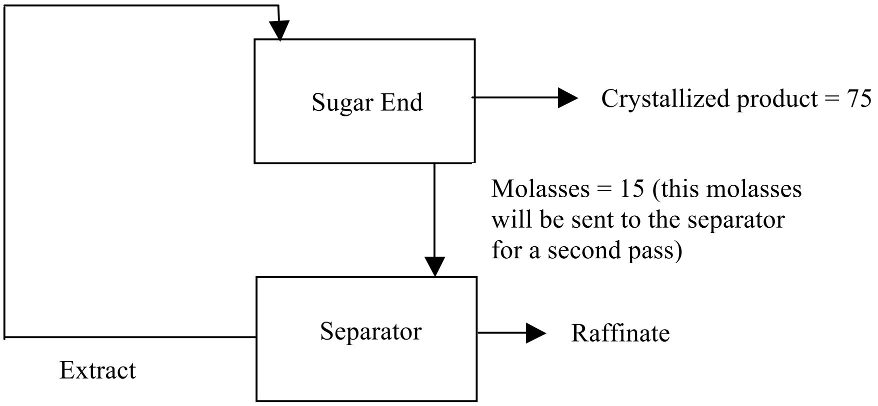
The Recycle Loop

The complete recycle configuration will be discussed in the following analysis. The results will be applicable to all other cases.

To observe the development of the recycle loop, it will be assumed that a portion of molasses can be labeled and then traced through the sugar end. In the following figure, 100 units of sucrose have been labeled in the molasses entering the separator. Across the separator, 90% sucrose recovery is assumed, so 90 units of labeled sucrose appear in the extract and 10 units in the raffinate. The labeled sucrose has not yet been recycled to the sugar end so crystallized product from this material is 0.



Next, the labeled extract enters the sugar end. Part of the extract sucrose is recovered as product and part is lost to molasses. Assuming 90 purity extract and 60 purity molasses, this will yield 75 units product and 15 units lost to molasses.



It is important to note that of the 100 units of sucrose entering the separator, only 75% was recovered as product. Additional labeled sucrose can be recovered by sending the 15 molasses units again into the separator—sugar end, separator— sugar end, etc. ad infinitum.

If it is assumed that the sugar end is in steady state (all internal recycle sugar and non-sugar loops built up to equilibrium), then the crystallized product recovered from a molasses separator is a function of:

1. Sucrose recovery across the separator.

2. Extract purity.

3. Molasses purity.

4. The number of times the entering material passes around the loop.

Functions

The first time the labeled sucrose passes through the separator-sugar end, the fractional amount of crystallized product recovered is given by:

$$a = R - (MRe / Em)$$

a = Fractional amount of crystallized product
R = Fractional separator recovery (example = 0.9)

M = Fractional molasses purity (example = 0.6)
E = Fractional extract purity (example = 0.9)
m = 1-M (example = 0.4)
e = 1-E (example = 0.1)

The fractional amount of sucrose lost to molasses after this first pass is given by:

$$r = MRe / Em$$

If the sucrose lost during the first pass, r, is sent again to the separator, the product obtained during this second pass is the entering sucrose r times the fraction recovery a. The sucrose lost in this second pass is the sucrose entering r times the fractional loss r. In table form:

Pass no. through the sugar end by the labeled sucrose	Fraction recovered as product this pass	Fraction lost to molasses this pass
1	a	r
2	ar	r^2
3	ar^2	r^3
.	.	.
.	.	.
.	.	.

Infinite Series

The above table is an infinite geometric series of constant terms.

$$a, ar, ar^2, ar^3, \ldots\ldots$$

The terms can be summed to yield the total sucrose recovered for any *n* number of passes:

$$\underset{\substack{\uparrow\\ \text{first pass}\\ \text{contribution}}}{a} + \underset{\substack{\uparrow\\ \text{second pass}\\ \text{contribution, etc.}}}{ar} + ar^2 + ar^3 + \ldots\ldots$$

As a frame of reference, it is also helpful to know the maximum amount of sucrose which can be recovered, i.e., assume the labeled sucrose recycles for an infinite number of passes.

This value can only be found if the infinite series is convergent. The requirement is:

If: $-1 < r < 1$

Then: $\sum_{k=o}^{\infty} ar^k = a/(1-r)$

The separator configuration can be demonstrated to be convergent as follows:

$$r = MRe / Em$$

All variables are positive so $-1 < r$. This satisfies part of the convergence requirement.

Furthermore, for a realistic system the extract purity is greater than the molasses purity:

$$E > M \quad \text{so} \quad \frac{M}{E} < 1$$

Also,

$$E > M$$
$$1 + E > 1 + M$$
$$1 - M > 1 - E$$
or
$m > e$ (since $m = 1\text{-}M$ and $e = 1\text{-}E$)

So,

$$\frac{e}{m} < 1$$

Recovery, R, is ≤ 1. So,

$$\left(\frac{M}{E}\right)\left(\frac{e}{m}\right)(R) < 1$$

Therefore, the infinite series will always be convergent.

Example 1: Fraction Sugar Recovered as Product
(This and the following example are at constant separator loading.)

Assume 60 purity molasses is produced. Two extract purity-recovery cases will be considered:

Separator		Pass 1	Pass 2	∞
Purity	Recovery			
90	**95**	**0.792**	**0.917……….**	**…….0.941**
95	**90**	**0.829**	**0.888……….**	**…….0.892**

The lower recovery, higher purity operation yields more product during the first pass than the high recovery operation. The higher recovery operation becomes favorable only after the second pass.

It is typical with a separator to gain extract purity at the expense of recovery. If an operator had the two above operating choices, he must consider the operating configuration. If the separator processes only fresh molasses and will not have any recycle, then the obvious choice is to operate the high purity condition (since there is only one pass available). This will yield the most product (82.9% sucrose as product versus 79.2%). Choosing the high purity operation is also correct if a separator has more feed molasses available than it can possibly treat in the operating time allotted. This choice will yield the most product for every day of operation. The high recovery conditions should never be chosen in this situation because in addition to low sucrose recovery, more non-sugar (color, etc.) is recycled to the process.

On the other hand, if there is a deficit of molasses for the operating time available, and a recycle or recycle plus discard operation is used, then it may be best to choose the high recovery, lower purity operation. This provides a chance to recover sucrose with a second pass, third pass, etc. This, of course, would also require a longer separator campaign period.

Note the sugar recovered at infinite passes. This is the maximum that can ever be recovered. It is always lower than the separator recovery because in every pass additional sucrose is lost to the raffinate.

Example 2: Fraction Sugar Recovered as Product

Again assume 60 purity molasses.

Separator		**Pass 1**	**Pass 2**	**∞**
Purity	**Recovery**			
85	**95**	**0.70**	**0.87……….**	**…….0.93**
95	**85**	**0.78**	**0.84……….**	**…….0.84**

In this example, the operator is aware of two possible separator purity-recovery conditions. As in Example 1, if this is a 100% fresh molasses operation or a large excess of molasses is available, it would be suggested that the high purity condition be implemented.

If this factory operates with excess time available then the higher recovery operation may be suggested. The reason the decision is not certain requires an understanding of the quantity of material input.

The material input for the first pass is proportional to one unit of entering sucrose. The second pass material input is proportional to r. The third proportional to r^2, etc.

Input load α $1 + r + r^3 + \ldots\ldots$

Here the product recovery table is again listed, but this time with the input load in parenthesis:

Separator		**Pass 1**	**Pass 2**	**∞**
Purity	**Recovery**			
85	**95**	**0.70(1)**	**0.87(1.23)……….**	**…….0.93(1.3)**
95	**85**	**0.78(1)**	**0.84(1.07)……….**	**…….0.84(1.07)**

Note that the low purity-high recovery condition, although recovering more product, requires about 12% more material input per unit recovered sucrose. This increased material input extends to material handling requirements, evaporation requirements, etc. Operating costs and material handling factors must be considered as part of the purity-recovery decision.

Product and Material Input Tables for 60 Purity Molasses

Two tables are provided at the end of this paper which list results for product sucrose recovery and material input. They are both based on production of 60 purity molasses. The first pass results apply to a single pass (fresh molasses) operation. The first pass through infinite pass results apply to a total recycle or recycle plus partial discard operation (for partial discard, multiply each number by the fractional quantity not discarded, for example, 0.8 for a 20% discard).

High Purity Separator Feed

Higher purity feed syrups, such as intermediate green, can be treated successfully through a chromatographic separator. These choices can have advantages such as elimination of low raw equipment. The same recycle equations apply. However it is important to note that as feed purity increases the amount of sucrose to be recovered rises dramatically. A very small loss in separator recovery can represent a substantial loss of sucrose in these cases. Material input is also very high relative to a molasses separator. For example, about twice as much water must eventually be evaporated if intermediate green is fed to the separator rather than molasses. The drawbacks increase as the separator is moved further towards the higher purity syrups.

Non-Sucrose Recycle

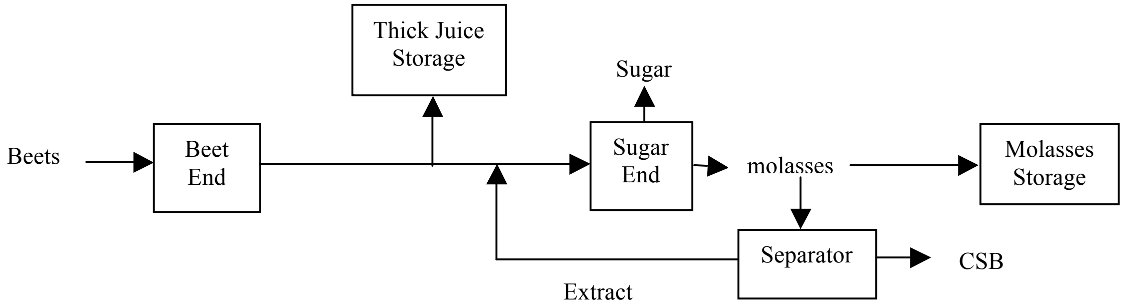
Although this paper has been restricted to the sucrose component of molasses, non-sucrose recycle is also of importance. Similar functions can easily be derived for evaluating this aspect of separator operation. Total recycle and partial discard configurations are of major concern in this case because the possibility exists for build-up of individual non-sucrose components.

Amalgamated Sugar's Twin Falls Separator Configurations

Amalgamated Sugar operates two of its own TASCO separators at its plant in Twin Falls, Idaho. Processing capacity is about 350 tons of 80% D.S. molasses per day. From recycle and other considerations, a program of variable operating configurations was developed.

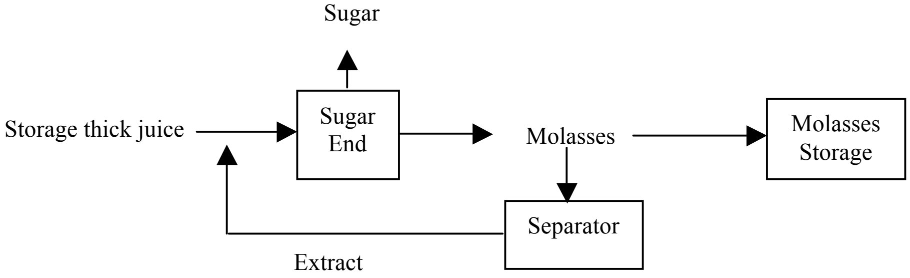
1. Beet campaign with separator – 148 days

This operation is implemented during the beet campaign. A partial molasses discard to storage is employed to prevent non-sucrose build-up. All extract is sent directly to the high melter. NOTE: CSB is concentrated separator by-product (the concentrated non-sucrose fraction from the separator).



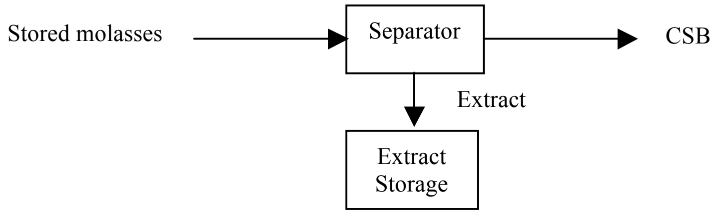
2. Thick juice campaign with separator – 26 days

During this period, stored thick juice is processed with the sugar end operating as during beet campaign.



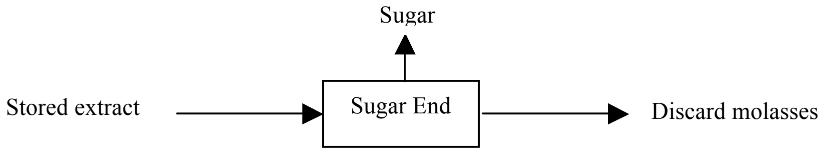
3. Molasses campaign with separator – 142 days

During this period the molasses stored (discarded) during beet and thick juice campaign is processed through the separator. The sugar end is not operated. All extract is stored.



4. Extract campaign – 15 days

During this period the stored extract from molasses campaign is sent through the sugar end. The sugar end operates on 100% extract (no thick juice). All molasses is discarded during this period to provide a final blow-down of poorly separated non-sucrose components. Extract campaign is followed by beet campaign.



Presented at the S.I.T. Annual Technical Meeting, Vancouver, British Columbia, Canada, May 6-9, 1990.

Fraction of Molasses Sucrose Recovered as Crystallized Product as a Function of Separator Recovery and Extract Purity

(molasses = 60 purity)

Extract Purity	Passes	Recovery across the Molasses Separator										
		100	98	96	94	92	90	88	86	84	82	80
100	1	1.000	0.980	0.960	0.940	0.920	0.900	0.880	0.860	0.840	0.820	0.800
	2	1.000	0.980	0.960	0.940	0.920	0.900	0.880	0.860	0.840	0.820	0.800
	3	1.000	0.980	0.960	0.940	0.920	0.900	0.880	0.860	0.840	0.820	0.800
	4	1.000	0.980	0.960	0.940	0.920	0.900	0.880	0.860	0.840	0.820	0.800
	5	1.000	0.980	0.960	0.940	0.920	0.900	0.880	0.860	0.840	0.820	0.800
	∞	1.000	0.980	0.960	0.940	0.920	0.900	0.880	0.860	0.840	0.820	0.800
98	1	0.969	0.950	0.931	0.911	0.892	0.872	0.853	0.834	0.814	0.795	0.776
	2	0.999	0.979	0.958	0.937	0.917	0.896	0.876	0.856	0.835	0.815	0.795
	3	1.000	0.979	0.959	0.938	0.918	0.897	0.877	0.856	0.836	0.815	0.795
	4	1.000	0.979	0.959	0.938	0.918	0.897	0.877	0.856	0.836	0.815	0.795
	5	1.000	0.979	0.959	0.938	0.918	0.897	0.877	0.856	0.836	0.815	0.795
	∞	1.000	0.979	0.959	0.938	0.918	0.897	0.877	0.856	0.836	0.815	0.795
96	1	0.938	0.919	0.900	0.881	0.863	0.844	0.825	0.806	0.788	0.769	0.750
	2	0.996	0.975	0.954	0.933	0.912	0.891	0.870	0.850	0.829	0.808	0.788
	3	1.000	0.978	0.957	0.936	0.915	0.894	0.873	0.852	0.831	0.810	0.789
	4	1.000	0.979	0.957	0.936	0.915	0.894	0.873	0.852	0.831	0.810	0.789
	5	1.000	0.979	0.957	0.936	0.915	0.894	0.873	0.852	0.831	0.810	0.789
	∞	1.000	0.979	0.957	0.936	0.915	0.894	0.873	0.852	0.831	0.810	0.789
94	1	0.904	0.886	0.868	0.850	0.832	0.814	0.796	0.778	0.760	0.741	0.723
	2	0.991	0.969	0.948	0.927	0.905	0.884	0.863	0.842	0.821	0.800	0.779
	3	0.999	0.977	0.955	0.933	0.912	0.890	0.868	0.847	0.826	0.804	0.783
	4	1.000	0.978	0.956	0.934	0.912	0.891	0.869	0.847	0.826	0.805	0.783
	5	1.000	0.978	0.956	0.934	0.912	0.891	0.869	0.847	0.826	0.805	0.783
	∞	1.000	0.978	0.956	0.934	0.912	0.891	0.869	0.847	0.826	0.805	0.783
92	1	0.870	0.852	0.835	0.817	0.800	0.783	0.765	0.748	0.730	0.713	0.696
	2	0.983	0.961	0.939	0.918	0.896	0.874	0.853	0.832	0.810	0.789	0.768
	3	0.998	0.975	0.952	0.930	0.908	0.885	0.863	0.841	0.819	0.797	0.776
	4	1.000	0.977	0.954	0.931	0.909	0.887	0.864	0.842	0.820	0.798	0.777
	5	1.000	0.977	0.954	0.932	0.909	0.887	0.864	0.842	0.820	0.798	0.777
	∞	1.000	0.977	0.954	0.932	0.909	0.887	0.864	0.842	0.820	0.798	0.777
90	1	0.833	0.817	0.800	0.783	0.767	0.750	0.733	0.717	0.700	0.683	0.667
	2	0.972	0.950	0.928	0.906	0.884	0.863	0.841	0.819	0.798	0.777	0.756
	3	0.995	0.972	0.948	0.925	0.902	0.879	0.857	0.834	0.812	0.789	0.767
	4	0.999	0.975	0.952	0.928	0.905	0.882	0.859	0.836	0.814	0.791	0.769
	5	1.000	0.976	0.952	0.929	0.905	0.882	0.859	0.837	0.814	0.791	0.769
	∞	1.000	0.976	0.952	0.929	0.906	0.882	0.859	0.837	0.814	0.792	0.769
88	1	0.795	0.780	0.764	0.748	0.732	0.716	0.700	0.684	0.668	0.652	0.636
	2	0.958	0.936	0.914	0.891	0.870	0.848	0.826	0.804	0.783	0.762	0.740
	3	0.991	0.967	0.943	0.919	0.895	0.872	0.849	0.826	0.803	0.780	0.758
	4	0.998	0.973	0.949	0.924	0.900	0.876	0.853	0.829	0.806	0.783	0.760
	5	1.000	0.975	0.950	0.925	0.901	0.877	0.853	0.830	0.807	0.784	0.761
	∞	1.000	0.975	0.950	0.926	0.901	0.877	0.854	0.830	0.807	0.784	0.761
86	1	0.756	0.741	0.726	0.710	0.695	0.680	0.665	0.650	0.635	0.620	0.605
	2	0.940	0.918	0.896	0.874	0.852	0.830	0.808	0.787	0.765	0.744	0.723
	3	0.985	0.960	0.936	0.911	0.887	0.863	0.839	0.815	0.792	0.769	0.746
	4	0.996	0.971	0.945	0.920	0.895	0.870	0.845	0.821	0.797	0.774	0.750
	5	0.999	0.973	0.947	0.922	0.896	0.871	0.847	0.822	0.798	0.775	0.751
	∞	1.000	0.974	0.948	0.922	0.897	0.872	0.847	0.823	0.799	0.775	0.751
84	1	0.714	0.700	0.686	0.671	0.657	0.643	0.629	0.614	0.600	0.586	0.571
	2	0.918	0.896	0.874	0.852	0.830	0.808	0.787	0.765	0.744	0.723	0.702
	3	0.977	0.951	0.925	0.900	0.875	0.851	0.826	0.802	0.779	0.755	0.732
	4	0.993	0.966	0.940	0.913	0.887	0.862	0.836	0.811	0.787	0.763	0.739
	5	0.998	0.971	0.943	0.917	0.890	0.864	0.839	0.814	0.789	0.764	0.740
	∞	1.000	0.972	0.945	0.918	0.891	0.865	0.840	0.814	0.789	0.765	0.741
82	1	0.671	0.657	0.644	0.630	0.617	0.604	0.590	0.577	0.563	0.550	0.537
	2	0.892	0.869	0.847	0.826	0.804	0.783	0.761	0.740	0.719	0.699	0.678
	3	0.964	0.938	0.912	0.886	0.861	0.836	0.811	0.786	0.762	0.739	0.715
	4	0.988	0.960	0.932	0.905	0.878	0.851	0.825	0.800	0.774	0.749	0.725
	5	0.996	0.967	0.939	0.911	0.883	0.856	0.829	0.803	0.778	0.752	0.725
	∞	1.000	0.970	0.942	0.913	0.885	0.858	0.831	0.805	0.779	0.753	0.725
80	1	0.625	0.613	0.600	0.588	0.575	0.563	0.550	0.538	0.525	0.513	0.500
	2	0.859	0.838	0.816	0.795	0.773	0.752	0.732	0.711	0.690	0.670	0.650
	3	0.947	0.920	0.894	0.868	0.842	0.816	0.791	0.767	0.742	0.719	0.695
	4	0.980	0.951	0.922	0.893	0.865	0.838	0.811	0.785	0.759	0.733	0.709
	5	0.993	0.962	0.932	0.902	0.874	0.845	0.818	0.791	0.764	0.738	0.713
	∞	1.000	0.968	0.938	0.907	0.878	0.849	0.821	0.793	0.766	0.740	0.714

Material Input to a Separator as a Function of Separator Recovery and Extract Purity

(molasses = 60 purity)

Extract Purity	Passes	Recovery across the Molasses Separator										
		100	98	96	94	92	90	88	86	84	82	80
100	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
98	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.031	1.030	1.029	1.029	1.028	1.028	1.027	1.026	1.026	1.025	1.024
	3	1.032	1.031	1.030	1.030	1.029	1.028	1.028	1.027	1.026	1.026	1.025
	4	1.032	1.031	1.030	1.030	1.029	1.028	1.028	1.027	1.026	1.026	1.025
	5	1.032	1.031	1.030	1.030	1.029	1.028	1.028	1.027	1.026	1.026	1.025
	∞	1.032	1.031	1.030	1.030	1.029	1.028	1.028	1.027	1.026	1.026	1.025
96	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.062	1.061	1.060	1.059	1.057	1.056	1.055	1.054	1.053	1.051	1.050
	3	1.066	1.065	1.064	1.062	1.061	1.059	1.058	1.057	1.055	1.054	1.052
	4	1.067	1.065	1.064	1.062	1.061	1.060	1.058	1.057	1.055	1.054	1.053
	5	1.067	1.065	1.064	1.062	1.061	1.060	1.058	1.057	1.055	1.054	1.053
	∞	1.067	1.065	1.064	1.062	1.061	1.060	1.058	1.057	1.055	1.054	1.053
94	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.096	1.094	1.092	1.090	1.088	1.086	1.084	1.082	1.080	1.079	1.077
	3	1.105	1.103	1.100	1.098	1.096	1.094	1.091	1.089	1.087	1.085	1.082
	4	1.106	1.103	1.101	1.099	1.097	1.094	1.092	1.090	1.087	1.085	1.083
	5	1.106	1.104	1.101	1.099	1.097	1.094	1.092	1.090	1.087	1.085	1.083
	∞	1.106	1.104	1.101	1.099	1.097	1.094	1.092	1.090	1.087	1.085	1.083
92	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.130	1.128	1.125	1.123	1.120	1.117	1.115	1.112	1.110	1.107	1.104
	3	1.147	1.144	1.141	1.138	1.134	1.131	1.128	1.125	1.122	1.118	1.115
	4	1.150	1.146	1.143	1.139	1.136	1.133	1.129	1.126	1.123	1.120	1.116
	5	1.150	1.147	1.143	1.140	1.136	1.133	1.130	1.126	1.123	1.120	1.116
	∞	1.150	1.147	1.143	1.140	1.136	1.133	1.130	1.126	1.123	1.120	1.117
90	1	1.100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.167	1.163	1.160	1.157	1.153	1.150	1.147	1.143	1.140	1.137	1.133
	3	1.194	1.190	1.186	1.181	1.177	1.172	1.168	1.164	1.160	1.155	1.151
	4	1.199	1.194	1.190	1.185	1.180	1.176	1.171	1.167	1.162	1.158	1.153
	5	1.200	1.195	1.190	1.186	1.181	1.176	1.172	1.167	1.163	1.158	1.154
	∞	1.200	1.195	1.190	1.186	1.181	1.176	1.172	1.167	1.163	1.158	1.154
88	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.205	1.200	1.196	1.192	1.188	1.184	1.180	1.176	1.172	1.168	1.164
	3	1.246	1.241	1.235	1.229	1.224	1.218	1.212	1.207	1.201	1.196	1.190
	4	1.255	1.249	1.242	1.236	1.230	1.224	1.218	1.212	1.206	1.201	1.195
	5	1.257	1.250	1.244	1.238	1.232	1.225	1.219	1.213	1.207	1.201	1.196
	∞	1.257	1.251	1.244	1.238	1.232	1.226	1.220	1.213	1.207	1.202	1.196
86	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.244	1.239	1.234	1.230	1.225	1.220	1.215	1.210	1.205	1.200	1.195
	3	1.304	1.297	1.289	1.282	1.275	1.268	1.261	1.254	1.247	1.240	1.234
	4	1.318	1.310	1.302	1.294	1.286	1.279	1.271	1.263	1.256	1.248	1.241
	5	1.322	1.314	1.305	1.297	1.289	1.281	1.273	1.265	1.258	1.250	1.242
	∞	1.323	1.315	1.306	1.298	1.290	1.282	1.274	1.266	1.258	1.250	1.243
84	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.286	1.280	1.274	1.269	1.263	1.257	1.251	1.246	1.240	1.234	1.229
	3	1.367	1.358	1.350	1.341	1.332	1.323	1.315	1.306	1.298	1.289	1.281
	4	1.391	1.380	1.370	1.360	1.350	1.340	1.331	1.321	1.311	1.302	1.293
	5	1.397	1.386	1.376	1.365	1.355	1.345	1.335	1.325	1.315	1.305	1.295
	∞	1.400	1.389	1.378	1.367	1.357	1.346	1.336	1.326	1.316	1.306	1.296
82	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.329	1.323	1.316	1.310	1.303	1.296	1.290	1.283	1.277	1.270	1.263
	3	1.438	1.427	1.416	1.405	1.395	1.384	1.374	1.363	1.353	1.343	1.333
	4	1.473	1.460	1.448	1.435	1.422	1.410	1.398	1.386	1.374	1.363	1.351
	5	1.485	1.471	1.458	1.444	1.431	1.418	1.405	1.392	1.380	1.368	1.356
	∞	1.491	1.476	1.462	1.448	1.435	1.421	1.408	1.395	1.382	1.370	1.358
80	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.375	1.367	1.360	1.352	1.345	1.337	1.330	1.322	1.315	1.307	1.300
	3	1.516	1.503	1.490	1.477	1.464	1.451	1.439	1.427	1.414	1.402	1.390
	4	1.568	1.552	1.536	1.521	1.505	1.490	1.475	1.460	1.445	1.431	1.417
	5	1.588	1.570	1.553	1.536	1.519	1.503	1.487	1.471	1.455	1.440	1.425
	∞	1.600	1.581	1.562	1.544	1.527	1.509	1.493	1.476	1.460	1.444	1.429